(Pages : 3) CCF(R) **A – 6482**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, October 2016
(2013 Scheme)
13.501 : ENGINEERING MATHEMATICS – IV (AFRT)
(Complex Analysis and Linear Algebra)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. Prove that the function f(z) = sin z is analytic and find its derivative.

2. For the conformal transformation $w = z^2$, find the coefficient of magnification and the angle of rotation at z = 1 + i.

3. Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the paths (a) y = x, (b) $y = x^2$.

4. Define a vector space. Give an example and justify that your example is a vector space.

5. Find the lengths and inner product of x = (1, 4, 0, 2) and y = (2, –2, 1, 3) with respect to the standard inner product of $\mathbb{R}^4$.

PART – B

Answer **any one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Show that an analytic function with constant modulus is constant.

   b) Show that a complex function which is analytic, at a point satisfies the Cauchy-Riemann equations.

   c) Discuss the transformation about $w = e^z$.

OR

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7. a) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$.

b) If $w = \phi + i\,\psi$ represents the complex potential for an electric field and

$\psi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, determine the function ϕ.

c) Under the transformation $w = \dfrac{1}{z}$, find the image of $|z - 2i| = 2$.

Module – II

8. a) Expand the function $f(z) = \cos z$ about $z = \dfrac{-\pi}{2}$ as Taylor's series.

b) Evaluate $\int_c \dfrac{z\,dx}{(z-1)(z-2)^2}$ where c is the circle $|z-2| = \dfrac{1}{2}$ using Cauchy's integral formula.

c) Show that $\int_0^{2z} \dfrac{\cos 2\theta\, d\theta}{1 - 2a\cos\theta + a^2} = \dfrac{2\pi a^2}{1 - a^2}, (a^2 < 1)$

OR

9. a) Evaluate $\int_c \dfrac{z+1}{z^2 + 2z + 4} dz$ where c is $|z + 1 + i| = 2$, using Cauchy's residue theorem.

b) Use Cauchy's integral formula to calculate $\int_c \dfrac{z^2 + 1}{z(2z+1)} dz$ where c is $|z| = 1$.

c) Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2 dx}{(x^2+1)(x^2+4)}$.

Module – III

10. a) Prove that if v_1, v_2, v_3 are linearly independent, then the vectors $w_1 = v_1 + v_2$, $w_2 = v_1 + v_3$, $w_3 = v_2 + v_3$ are also linearly independent.

b) Describe the column space and null space of the matrix $A=\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

OR

11. a) Find the linear transformation $T : R^2 \rightarrow R^3$ which takes (1, 2) to (3, 2, 1) and (3, 4) to (6, 5, 4).

b) Examine whether the vectors (1, 2, 3), (1, –1, 1) and (1, 0, 1) form a basic of $\mathbb{R}^3$.

Module – IV

12. a) Find a vector x orthogonal to the rowspace and a vector y orthogonal to the column space of $A=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$.

b) Find a maxima or minima of $5x_1^2+5x_2^2+4x_1x_2$ subject to $X^T X = 1$ where $X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

OR

13. Use Gram-Schmidt orthogonalisation to generate a set of orthonormal vectors from $a_1=\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, a_2=\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $a_3=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
